

STREAMING CAUSED BY IMPULSE ACOUSTIC WAVE.

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Abstract

Five eigenvectors of the linear thermoviscous flow over the homogeneous background derived for the quasi-plane geometry of the flow. The corresponding projectors are calculated and applied to get the nonlinear evolution equations for the interacting vortical and acoustic modes. Equation on streaming caused by arbitrary acoustic wave is specified. The correspondence to the known results on streaming caused by quasi-periodic source is traced. The radiation acoustic force is calculated for the mono-polar source.

1 INTRODUCTION

Traditionally, streaming is referred as a localized mean flow caused by an acoustic source. This flow is created by the Reynolds forces, non-zero time-averaged values of the quadratic acoustic terms arising in the equation of conservation of momentum. There exist extensive reviews on this subject [1-3].

Steady acoustic streaming is described by the mean Eulerian velocity $\langle \vec{v} \rangle$, or the mean mass flow velocity \vec{U} , where \vec{U} is defined by $\vec{U} = \frac{\langle \rho \vec{v} \rangle}{\rho_0} = \vec{V} + \frac{\langle \rho_a \vec{v}_a \rangle}{\rho_0}$. Here, \vec{V} is a slow component of velocity, namely the Eulerian velocity of the streaming, index a labels acoustic wave, ρ_0 is undisturbed density. Square brackets designate averaging over time whose interval is much shorter than the transient time of streaming and much longer than a period of a sound wave. It is pointed, that the difference between \vec{U} and \vec{V} is roughly proportional to the acoustic intensity vector and is small compared with any of mean velocities [4]. From the continuity equation it follows $\nabla \vec{U} = 0$ if we assume $\left\langle \frac{\partial \rho}{\partial t} \right\rangle = 0$.

The recent discussion of acoustic streaming [5,6] suggest that there is important unresolved issue concerning acoustic streaming, namely the effect of fluid incompressibility. The inconsistency of this point is that fluid is supposed to be incompressible while the acoustic wave causing streaming may propagate only over compressible medium. The next, the conservation of energy usually is not considered. It is reasonable since excluding of one variable (density) reduces an initial system. It has been proved that the effect of the heat conductivity could not be discarded in a study of temperature variation associated with the streaming [6]. Actually, this approximation is well understood in a typical liquid like water but should be revised for other liquids [7]. That is a reason to start from the full system of conservation laws including the energy balance equation and the continuity equation.

The procedure of the temporal averaging needs a continuous acoustic wave of many periods as a source, thus the well-known results on acoustic streaming apply only for this type of acoustic source though quite realistic. There are clear inconsistency concerning the very procedure of the temporal averaging. As it has been mentioned before, an equality $\nabla \vec{U} = 0$ is correct only if $\left\langle \frac{\partial \rho}{\partial t} \right\rangle = 0$. Actually, the overall field includes not only acoustic and vortical modes, but also of the entropy mode. The entropy mode caused by acoustic wave is known as a slow decrease of density and isobaric increase of temperature of the background of acoustic wave propagation. Recently, the importance of change in density of surrounding (not only increase in temperature) was underlined in the review [8]. If nonlinear interactions are accounted in a proper way, for density of the entropy mode one gets an equation :

$$\frac{\partial \rho_e}{\partial t} + \frac{\eta}{\rho_0} \Delta \rho_e = S_a,$$

where η is shear viscosity, S_a is an acoustic source proportional to a gradient of acoustic intensity, index e concerns the entropy mode. An attenuation of acoustic wave leads to slow but continuous decrease of density so $\langle \frac{\partial \rho_e}{\partial t} \rangle = 0$ becomes incorrect as well as $\langle \frac{\partial \rho}{\partial t} \rangle = 0$, since $\rho = \rho_0 + \rho_a + \rho_e$. Therefore, if the acoustic heating caused by periodic acoustic wave is accounted, $\nabla \vec{U} = 0$ becomes incorrect.

These inconsistencies call new ways to theoretical investigation of acoustic streaming, including streaming caused by non-periodic acoustic wave. We plan to start with all conservation equations for compressible thermoviscous flow, then to define modes by establishing of the relations between perturbations (section 2), to demonstrate how the well-known linear and nonlinear evolution equations may be obtained by projecting (sections 3,4), and, finally, to apply projectors to calculating of the streaming caused by pulse acoustic wave (section 5).

The idea to decompose the linear flow into specific modes is not novel and has been exploited for a long time, see [9] and referred papers, where homogeneous background with sources of heat and mass are considered. The concrete ideas of doing it automatically by using of projectors in the wide variety of problems (also flow over inhomogeneous media like bubbly liquid [10], flows affected by external forces including the gravitational one which changes the background density and pressure [11], both one- and multi-dimensional problems, see [12]). The principle advance is an expansion of the ideas into area of nonlinear flow: to get nonlinear coupled evolution equations for interacting modes and to solve the system approximately. The deriving of the final nonlinear system is algorithmic, one just acts by projectors at the initial system of nonlinear equations. Some ideas concerning separation and interaction of acoustic-gravity and Rossby waves may be found in [13].

2 Modes and projectors for the flow with equations of state in the general form.

The mass, momentum and energy conservation equations for the thermoviscous flow read:

$$\begin{aligned} \frac{\partial \rho}{\partial t} + \vec{\nabla}(\rho \vec{v}) &= 0 \\ \rho \left[\frac{\partial \vec{v}}{\partial t} + (\vec{v} \vec{\nabla}) \vec{v} \right] &= -\vec{\nabla} p + \eta \Delta \vec{v} + \left(\varsigma + \frac{\eta}{3} \right) \vec{\nabla}(\vec{\nabla} \vec{v}) \\ \rho \left[\frac{\partial e}{\partial t} + (\vec{v} \vec{\nabla}) e \right] + p \vec{\nabla} \vec{v} - \chi \Delta T &= \varsigma (\vec{\nabla} \vec{v})^2 + \frac{\eta}{2} \left(\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i} - \frac{2}{3} \delta_{ik} \frac{\partial v_l}{\partial x_l} \right)^2 \end{aligned} \quad (1)$$

Here, among already mentioned variables, p is pressure, e, T - internal energy per unit mass and temperature; ς, χ are bulk viscosity and thermal conductivity coefficient (both supposed to be constants as well as η), x_i - space coordinates. Except of the dynamic equations (1), the two thermodynamic relations are necessary: $e(p, \rho), T(p, \rho)$. To treat a wide variety of substances, let us use the most general form of these relations as expansion in the Fourier series:

$$\begin{aligned} \rho_0 e' &= E_1 p' + \frac{E_2 p_0}{\rho_0} \rho' + \frac{E_3}{p_0} p'^2 + \frac{E_4 p_0}{\rho_0^2} \rho'^2 + \frac{E_5}{\rho_0} p' \rho' + \dots \\ T' &= \frac{\Theta_1}{\rho_0 C_v} p' + \frac{\Theta_2 p_0}{\rho_0^2 C_v} \rho' + \dots \end{aligned} \quad (2)$$

The background values are marked by zero, perturbations are primed, C_v means specific heat per unit mass at constant volume, $E_1, \dots, \Theta_1, \dots$ are dimensionless coefficients,

$$\Theta_1 = \frac{\rho_0 C_v k}{\beta}, \quad \Theta_2 = -\frac{\rho_0 C_v \beta}{p_0}, \quad (3)$$

where $k = \frac{1}{\rho_0} \left(\frac{\partial p}{\partial \rho} \right)_{T=T(p_0, \rho_0)}$, $\beta = -\frac{1}{\rho_0} \left(\frac{\partial \rho}{\partial T} \right)_{p=p_0}$.

For the three-dimensional flow $\vec{v} = (v_x, v_y, v_z)$ in the Cartesian coordinates $\vec{x} = (x, y, z)$ over homogeneous stationary background without mean flow $\vec{v}_0 = 0$, the system (1), (2) looks:

$$\rho_0 \frac{\partial v_x}{\partial t} + \frac{\partial p}{\partial x} - \eta \Delta v_x - \left(\varsigma + \frac{\eta}{3} \right) \frac{\partial}{\partial x} (\vec{\nabla} v) = -\rho_0 (\vec{v} \vec{\nabla}) v_x + \frac{\rho'}{\rho_0} \frac{\partial p}{\partial x} - \eta \rho' \Delta v_x - \left(\varsigma + \frac{\eta}{3} \right) \rho' \frac{\partial}{\partial x} (\vec{\nabla} v)$$

$$\rho_0 \frac{\partial v_y}{\partial t} + \frac{\partial p}{\partial y} - \eta \Delta v_y - \left(\varsigma + \frac{\eta}{3} \right) \frac{\partial}{\partial y} (\vec{\nabla} v) = -\rho_0 (\vec{v} \vec{\nabla}) v_y + \frac{\rho'}{\rho_0} \frac{\partial p}{\partial y} - \eta \rho' \Delta v_y - \left(\varsigma + \frac{\eta}{3} \right) \rho' \frac{\partial}{\partial y} (\vec{\nabla} v)$$

$$\rho_0 \frac{\partial v_x}{\partial t} + \frac{\partial p}{\partial x} - \eta \Delta v_x - \left(\varsigma + \frac{\eta}{3} \right) \frac{\partial}{\partial x} (\vec{\nabla} v) = -\rho_0 (\vec{v} \vec{\nabla}) v_x + \frac{\rho'}{\rho_0} \frac{\partial p}{\partial x} - \eta \rho' \Delta v_x - \left(\varsigma + \frac{\eta}{3} \right) \rho' \frac{\partial}{\partial x} (\vec{\nabla} v)$$

$$\frac{\partial p'}{\partial t} + c^2 \rho_0 \vec{\nabla} \vec{v} - \frac{\chi}{E_1} \left(\frac{\Theta_1}{\rho_0 C_v} \Delta p' + \frac{\Theta_2 p_0}{\rho_0^2 C_v} \Delta \rho' \right) = \frac{1}{E_1} \left(\varsigma (\vec{\nabla} \vec{v})^2 + \frac{\eta}{2} \left(\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i} - \frac{2}{3} \delta_{ik} \frac{\partial v_l}{\partial x_l} \right)^2 \right) + (Q p' + S \rho') \vec{\nabla} \vec{v} - \vec{v} \vec{\nabla} p$$

$$\frac{\partial \rho'}{\partial t} + \rho_0 \vec{\nabla} \vec{v} = -\rho' \vec{\nabla} \vec{v} - \vec{v} \vec{\nabla} \rho' \quad (4),$$

where Q,S are constants depending on the equations of state (2) :

$$Q = \frac{1}{E_1} (-1 + 2 \frac{1-E_2}{E_1} E_3 + E_5),$$

$$S = \frac{1}{1-E_2} (1 + E_2 + 2E_4 + \frac{1-E_2}{E_1} E_5). \quad (5)$$

For an ideal gas, $Q = -\gamma = -C_P/C_v$, $S = 0$.

In order to simplify calculations, the geometry of beams will be considered. The equivalent system in the dimensionless variables :

$$\vec{v}_*, \vec{x}_*, \rho_*, p_*, t_* : \vec{v} = \varepsilon c \vec{v}_*, p' = \varepsilon c^2 \rho_0 p_*, \rho' = \varepsilon \rho_0 \rho_*, \vec{x} = (\lambda x_*/\sqrt{\mu}, \lambda y_*/\sqrt{\mu}, \lambda z_*/\sqrt{\mu}), t = \lambda t_*/c, \quad (6)$$

($c = \sqrt{\frac{p_0(1-E_2)}{\rho_0 E_1}}$ is adiabatic sound velocity, λ means characteristic scale of the longitudinal perturbations, μ is small parameter expressing the relation the longitudinal and transverse scales of perturbation, ε is small amplitude parameter) looks as follows (asterisks for dimensionless variables will be later omitted):

$$\frac{\partial}{\partial t} \psi + L \psi = \varepsilon \varphi + \varepsilon \varphi_{tv}, \quad (7)$$

where ψ is column of field perturbations

$$\psi = \begin{pmatrix} v_x & v_y & v_z & p & \rho \end{pmatrix}^T, \quad (8)$$

L is a linear matrix operator :

$$L = \begin{pmatrix} -\delta_1^1 \mu \frac{\partial^2}{\partial x^2} - \delta_1^2 \Delta & -\delta_1^1 \sqrt{\mu} \frac{\partial^2}{\partial x \partial y} & -\delta_1^1 \mu \frac{\partial^2}{\partial x \partial z} & \sqrt{\mu} \partial / \partial x & 0 \\ -\delta_1^1 \sqrt{\mu} \frac{\partial^2}{\partial x \partial y} & -\delta_1^1 \frac{\partial^2}{\partial y^2} - \delta_1^2 \Delta & -\delta_1^1 \sqrt{\mu} \frac{\partial^2}{\partial y \partial z} & \partial / \partial y & 0 \\ -\delta_1^1 \mu \frac{\partial^2}{\partial x \partial z} & -\delta_1^1 \sqrt{\mu} \frac{\partial^2}{\partial z \partial y} & -\delta_1^1 \mu \frac{\partial^2}{\partial z^2} - \delta_1^2 \Delta & \sqrt{\mu} \partial / \partial z & 0 \\ \sqrt{\mu} \partial / \partial x & \partial / \partial y & \sqrt{\mu} \partial / \partial z & -\delta_2^1 \Delta & -\delta_2^2 \Delta \\ \sqrt{\mu} \partial / \partial x & \partial / \partial y & \sqrt{\mu} \partial / \partial z & 0 & 0 \end{pmatrix} \quad (9)$$

with dimensionless parameters

$$\delta_1^1 = \frac{(\varsigma + \eta/3)}{\rho_0 c \lambda}, \delta_1^2 = \frac{\eta}{\rho_0 c \lambda}, \delta_2^1 = \frac{\chi \theta_1}{\rho_0 c \lambda_0 C_v E_1}, \delta_2^2 = \frac{\chi \theta_2}{\rho_0 c \lambda_0 C_v (1-E_2)}.$$

Dimensionless operators $\vec{\nabla}, \Delta$ look : $\vec{\nabla} = \begin{pmatrix} \mu \partial / \partial x & \partial / \partial y & \mu \partial / \partial z \end{pmatrix}$, $\Delta = \mu \partial^2 / \partial x^2 + \partial^2 / \partial y^2 + \mu \partial^2 / \partial z^2$, φ is the quadratic nonlinear column:

$$\varphi = \begin{pmatrix} -(\vec{v} \vec{\nabla}) v_x + \sqrt{\mu} \rho \partial p / \partial x \\ -(\vec{v} \vec{\nabla}) v_y + \rho \partial p / \partial y \\ -(\vec{v} \vec{\nabla}) v_z + \sqrt{\mu} \rho \partial p / \partial z \\ [Q p + S \rho] (\vec{\nabla} \vec{v}) - (\vec{v} \vec{\nabla}) p \\ -\rho (\vec{\nabla} \vec{v}) - (\vec{v} \vec{\nabla}) \rho \end{pmatrix} \quad (10),$$

and φ_{tv} is the quadratic nonlinear column $O(\beta)$ appearing in the viscous flow:

$$\varphi_{tv} = \begin{pmatrix} -\delta_1^2 \rho \Delta v_x - \delta_1^1 \rho \frac{\partial}{\partial x} (\vec{\nabla} v) \\ -\delta_1^2 \rho \Delta v_y - \delta_1^1 \rho \frac{\partial}{\partial y} (\vec{\nabla} v) \\ -\delta_1^2 \rho \Delta v_z - \delta_1^1 \rho \frac{\partial}{\partial z} (\vec{\nabla} v) \\ \frac{1}{E_1} \left((\delta_1^1 - \delta_1^2/3) (\vec{\nabla} v)^2 + \frac{\delta_2^2}{2} \left(\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i} - \frac{2}{3} \delta_{ik} \frac{\partial v_l}{\partial x_l} \right)^2 \right) \\ 0 \end{pmatrix} \quad (11)$$

For a linear flow defined by the linearized version of system (7)

$$\frac{\partial}{\partial t} \psi + L\psi = 0, \quad (12)$$

a solution may be found as a sum of planar waves: $v_x = \tilde{v}_x(\vec{k}) \exp(i\omega t - i\vec{k} \cdot \vec{x})$, ...where

$$\vec{k} = (k_x, k_y, k_z)$$

is the wave vector. It is convenient to go to the Fourier transforms marked by tilde for the flows over the homogeneous background. In the Fourier space, $-ik_x$ means $\partial/\partial x$, $i\omega$ means $\partial/\partial t$, and (6) yields in the five roots of dispersion relation ($\beta = \delta_1^1 + \delta_1^2 + \delta_2^1 + \delta_2^2$):

$$\omega_1 = \Omega + i\beta\Omega^2/2, \quad \omega_2 = -\Omega + i\beta\Omega^2/2, \quad \omega_3 = -i\delta_2^2\Omega^2, \quad \omega_4 = i\delta_1^2\Omega^2, \quad \omega_5 = i\delta_1^2\Omega^2, \text{ where}$$

$$\Omega = k_y + \frac{\mu(k_x^2 + k_z^2)}{2k_y}. \quad (13)$$

Also, an operator $(-ik_y)^{-1}$ represents in \vec{k} -space operator $\int dy$. Constant of integration should be chosen accordingly to the concrete physical problem. The first two roots relate to progressive (acoustic) modes of different directions of propagation, the third one relates to the entropy mode, and the fourth and fifth ones - to the vortical ones. For the real substances, $\beta > 0$, and, $\delta_2^2 < 0$, that corresponds physically correct sign of imaginary parts of all frequencies. The modes of linear flow are determined by relations of amplitudes of plane waves $\tilde{v}_x(k_x, k_y, k_z)$, ... Modes as eigenvectors of a linear problem in the Fourier space look:

$$\begin{aligned} \tilde{\psi}_1 = \begin{pmatrix} \tilde{v}_{x1}(k_x, k_y, k_z) \\ \tilde{v}_{y1}(k_x, k_y, k_z) \\ \tilde{v}_{z1}(k_x, k_y, k_z) \\ \tilde{p}_1(k_x, k_y, k_z) \\ \tilde{\rho}_1(k_x, k_y, k_z) \end{pmatrix} &= \begin{pmatrix} 1 - \mu(k_x^2 + k_z^2)/(2k_y^2) + i\beta k_y/2 \\ \sqrt{\mu} k_x/k_y \\ \sqrt{\mu} k_z/k_y \\ 1 + i(\delta_2^1 + \delta_2^2)k_y \\ 1 \end{pmatrix} \tilde{\rho}_1 = \tilde{M}_1 \tilde{\rho}_1, \\ \tilde{\psi}_2 = \begin{pmatrix} -\sqrt{\mu} k_x/k_y \\ -1 + \mu(k_x^2 + k_z^2)/(2k_y^2) + i\beta k_y/2 \\ -\sqrt{\mu} k_z/k_y \\ 1 - i(\delta_2^1 + \delta_2^2)k_y \\ 1 \end{pmatrix} \tilde{\rho}_2 = \tilde{M}_2 \tilde{\rho}_2, \\ \tilde{\psi}_3 = \begin{pmatrix} 0 \\ -i\delta_2^2 k_y \\ 0 \\ 0 \\ 1 \end{pmatrix} \tilde{\rho}_3 = \tilde{M}_3 \tilde{\rho}_3, \quad \tilde{\psi}_4 = \begin{pmatrix} ik_y \\ -i\sqrt{\mu} k_x \\ 0 \\ 0 \\ 0 \end{pmatrix} \tilde{\varphi}_4 = \tilde{M}_4 \tilde{\varphi}_4, \quad \tilde{\psi}_5 = \begin{pmatrix} 0 \\ -i\sqrt{\mu} k_z \\ ik_y \\ 0 \\ 0 \end{pmatrix} \tilde{\varphi}_5 = \tilde{M}_5 \tilde{\varphi}_5. \end{aligned} \quad (14)$$

All calculations of modes and projectors have accuracy up to the terms of order μ, β . As basic variable for the first three modes the perturbation of density is chosen, and for the last two- the stream function, since for the both vorticity modes density keeps unperturbed. So, there are five independent values indeed, one for every mode, that define other wave perturbations of the mode uniquely. Any field of the linear flow as a solution of a linearized equation (12) may be presented as a sum of independent modes.

The next step is to get projectors that decompose a concrete mode from the overall field $\tilde{\psi}$. Let us define a matrix M in the following way:

$$\widetilde{M} \begin{pmatrix} \widetilde{\rho}_1 \\ \widetilde{\rho}_2 \\ \widetilde{\rho}_3 \\ \widetilde{\varphi}_4 \\ \widetilde{\varphi}_5 \end{pmatrix} = \widetilde{\psi} = \sum_{i=1}^5 \widetilde{\psi}_i = \widetilde{M}_1 \widetilde{\rho}_1 + \widetilde{M}_2 \widetilde{\rho}_2 + \widetilde{M}_3 \widetilde{\rho}_3 + \widetilde{M}_4 \widetilde{\varphi}_4 + \widetilde{M}_5 \widetilde{\varphi}_5. \quad (15)$$

so that matrix \widetilde{M} consists of five columns,

$$\widetilde{M} = (\widetilde{M}_1 \quad \widetilde{M}_2 \quad \widetilde{M}_3 \quad \widetilde{M}_4 \quad \widetilde{M}_5). \quad (16)$$

and the inverse matrix \widetilde{M}^{-1} consists of five lines

$$\widetilde{M}^{-1} = \begin{pmatrix} \widetilde{M}_1^{-1} & \widetilde{M}_2^{-1} & \widetilde{M}_3^{-1} & \widetilde{M}_4^{-1} & \widetilde{M}_5^{-1} \end{pmatrix}^T,$$

where lines $\widetilde{M}_1^{-1}, \dots, \widetilde{M}_5^{-1}$ are as follows:

$$\begin{aligned} \widetilde{M}_1^{-1} &= \begin{pmatrix} \sqrt{\mu} \frac{k_x}{k_y} & \frac{1}{2} (1 - i(\delta_2^1 + \delta_2^2)k_y - \mu(k_x^2 + k_z^2)/(2k_y)) & \sqrt{\mu} \frac{k_z}{k_y} & \frac{1}{2} (1 - i\beta k_y/2 - i\delta_2^2 k_y) & i\delta_2^2 k_y/2 \end{pmatrix}, \\ \widetilde{M}_2^{-1} &= \begin{pmatrix} -\sqrt{\mu} \frac{k_x}{k_y} & \frac{1}{2} (-1 - i(\delta_2^1 + \delta_2^2)k_y + \mu(k_x^2 + k_z^2)/(2k_y)) & -\sqrt{\mu} \frac{k_z}{k_y} & \frac{1}{2} (1 + i\beta k_y/2 + i\delta_2^2 k_y) & -i\delta_2^2 k_y/2 \end{pmatrix}, \\ \widetilde{M}_3^{-1} &= \begin{pmatrix} 0 & \frac{1}{2} i(\delta_2^1 + \delta_2^2)k_y & 0 & -1 & 1 \end{pmatrix}, \\ \widetilde{M}_4^{-1} &= \begin{pmatrix} i \frac{1 - \mu k_x^2/k_y^2}{k_y} & -i\sqrt{\mu} \frac{k_x}{k_y^2} & -i\mu \frac{k_x k_z}{k_y^3} & 0 & 0 \end{pmatrix}, \\ \widetilde{M}_5^{-1} &= \begin{pmatrix} -i\mu \frac{k_x k_z}{k_y^3} & -i\sqrt{\mu} \frac{k_z}{k_y^2} & i \frac{1 - \mu k_z^2/k_y^2}{k_y} & 0 & 0 \end{pmatrix}. \end{aligned} \quad (17)$$

Accordingly to definition of matrix \widetilde{M}^{-1} projectors may be determined, so as

$$\widetilde{P}_1 \widetilde{\psi} = \widetilde{\psi}_1, \dots, \widetilde{P}_5 \widetilde{\psi} = \widetilde{\psi}_5. \quad (18)$$

Relations (14), (15) written for the first mode as an example $\widetilde{M}_1^{-1} \widetilde{\psi} = \rho_1, \widetilde{\psi}_1 = \widetilde{M}_1 \rho_1$ lead to $\widetilde{P}_1 = \widetilde{M}_1 \cdot \widetilde{M}_1^{-1}$ accordingly to (18) and so on for the all other modes:

$$\widetilde{P}_i = \widetilde{M}_i \cdot \widetilde{M}_i^{-1}, i = 1, \dots, 5. \quad (19)$$

At any moment of evolution a concrete mode is distinguished from the overall field by the correspondent projector. Projectors calculated with accuracy of order μ, β look:

$$\begin{aligned} \widetilde{P}_{1,2} &= \begin{pmatrix} \mu \frac{k_x^2}{2k_y^2} & \sqrt{\mu} \frac{k_x}{2k_y} & \mu \frac{k_x k_z}{2k_y^2} & \pm \sqrt{\mu} \frac{k_x}{2k_y} & 0 \\ \sqrt{\mu} \frac{k_x}{2k_y} & \frac{1}{2} \left(1 \pm \frac{i\beta}{2} k_y \mp i(\delta_2^1 + \delta_2^2)k_y - \mu \frac{k_x^2 + k_z^2}{2k_y^2} \right) & \sqrt{\mu} \frac{k_z}{2k_y} & \frac{1}{2} \left(\pm 1 - i\delta_2^2 k_y - \mu \frac{k_x^2 + k_z^2}{2k_y^2} \right) & \frac{i\delta_2^2 k_y}{2} \\ \mu \frac{k_x k_z}{2k_y^2} & \sqrt{\mu} \frac{k_z}{2k_y} & \mu \frac{k_z^2}{2k_y^2} & \pm \sqrt{\mu} \frac{k_z}{2k_y} & 0 \\ \pm \sqrt{\mu} \frac{k_x}{2k_y} & \pm \frac{1}{2} \left(1 - \mu \frac{k_x^2 + k_z^2}{2k_y^2} \right) & \pm \sqrt{\mu} \frac{k_z}{2k_y} & \frac{1}{2} (1 \mp \frac{i\beta}{2} k_y \pm i\delta_2^1 k_y) & \pm \frac{i\delta_2^2 k_y}{2} \\ \pm \sqrt{\mu} \frac{k_x}{2k_y} & \frac{1}{2} \left(\pm 1 - i(\delta_2^1 + \delta_2^2)k_y \mp \mu \frac{k_x^2 + k_z^2}{2k_y^2} \right) & \pm \sqrt{\mu} \frac{k_z}{2k_y} & \frac{1}{2} (1 \mp \frac{i\beta}{2} k_y \mp i\delta_2^2 k_y) & \pm \frac{i\delta_2^2 k_y}{2} \end{pmatrix}, \\ \widetilde{P}_3 &= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & i\delta_2^2 k_y & -i\delta_2^2 k_y \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & i(\delta_2^1 + \delta_2^2)k_y & 0 & -1 & 1 \end{pmatrix}, \\ \widetilde{P}_4 &= \begin{pmatrix} 1 - \mu \frac{k_x^2}{k_y^2} & -\sqrt{\mu} \frac{k_x}{k_y} & -\mu \frac{k_x k_z}{k_y^2} & 0 & 0 \\ -\sqrt{\mu} \frac{k_x}{k_y} & \mu \frac{k_x^2}{k_y^2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad \widetilde{P}_5 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \mu \frac{k_z^2}{k_y^2} & -\sqrt{\mu} \frac{k_z}{k_y} & 0 & 0 \\ -\mu \frac{k_x k_z}{k_y^2} & \sqrt{\mu} \frac{k_z}{k_y} & 1 - \mu \frac{k_z^2}{k_y^2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \end{aligned} \quad (20)$$

Matrix projectors satisfy common properties of orthogonal projectors:

$$\sum_{i=1}^5 \widetilde{P}_i = \widetilde{I}, \widetilde{P}_i \cdot \widetilde{P}_n = \widetilde{0} \text{ if } i \neq n, \widetilde{P}_i \cdot \widetilde{P}_i = \widetilde{P}_i \text{ if } i = n, \quad (21)$$

where $\widetilde{I}, \widetilde{0}$ are unit and zero matrices. The inverse transformation of formulae (20) to the (\vec{x}, t) space may be easily undertaken.

3 Linear flow and evolution equations.

A linear flow may be decomposed to modes uniquely accordingly to (18). Linear evolution equations for every mode may be originated from the linear version of (7) by acting of the corresponding projector: $P(\frac{\partial}{\partial t}\psi + L\psi) = 0$. Note that all projectors do commute with both $\partial/\partial t \cdot I$ and L (I is unit matrix). When one acts by projector at the linear system (12), five equations for all the components of every mode appear. For example, an evolution equation for the dimensionless perturbations of density for both acoustic modes are as follows:

$$\frac{\partial \rho_{1,2}}{\partial t} \pm \frac{\partial \rho_{1,2}}{\partial y} \pm \frac{\mu}{2} \int \Delta_{\perp} \rho_{1,2} dy - \frac{\beta}{2} \frac{\partial^2 \rho_{1,2}}{\partial y^2} = 0, \quad (22)$$

$$\text{where } \Delta_{\perp} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}.$$

An evolution equation for the axial velocity of the entropy mode looks:

$$\frac{\partial v_{y3}}{\partial t} - \delta_2^2 \frac{\partial^2 v_{y3}}{\partial y^2} = 0, \quad (23)$$

and for the axial velocity of two vortical modes is:

$$\frac{\partial v_{y4,5}}{\partial t} - \delta_1^2 \frac{\partial^2 v_{y4,5}}{\partial y^2} = 0. \quad (24)$$

These linear evolution equations are well-known. Equations (21)-(23) are explicit up to terms of order $O(\beta\sqrt{\mu}, \mu^2)$ due to accuracy of evaluated projectors. In the paper [9], equations (22)-(24) are presented as three equations for the three basic modes, since there was no subdivision of acoustic and vortical modes into two independent branches everyone. Also, the evolution equations from [9] involves sources of mass, momentum and heat that easily may be added in to the right-hand side of the initial system (7) as well as to the equations (22)-(24).

4 Nonlinear flow and coupled evolution equations.

At this point, we declare that relations inside every mode become fixed in weakly nonlinear flow. Acting by projectors at the original system (7) with non-zero nonlinear part leads to coupled equations for interacting modes: $P(\frac{\partial}{\partial t}\psi + L\psi) = \varepsilon P(\varphi_1 + \varphi_{1tv})$. Modes become separated in the linear side, and one should account that the overall field is a sum of all modes inputs in the nonlinear side. The proper returning to the \vec{x} space should be proceeded as well. To illustrate the possibilities of the projecting method, some illustrations concerning the famous evolution equations deriving are presented below.

4.1 One-dimensional nonlinear flow.

Let $k_x = 0$, $k_z = 0$, that corresponds to one-dimensional flow along y-axis. For the planar geometry, only three modes exist: two acoustic and the entropy one. Acting by P_1 at the system (5), one get an evolution equation for the rightward acoustic mode consisting of three equations for p_1 , ρ_1 , and v_1 . An evolution equation for the non-dimensional density perturbations of the rightward acoustic mode is as follows:

$$\frac{\partial \rho_1}{\partial t} + \frac{\partial \rho_1}{\partial y} - \frac{\beta}{2} \frac{\partial^2 v_1}{\partial y^2} = \frac{\varepsilon}{2} \left(-v \frac{\partial}{\partial y} v + \rho \frac{\partial}{\partial y} p - v \frac{\partial}{\partial y} p + [Qp + S\rho] \frac{\partial}{\partial y} v \right) \quad (25),$$

Here, p , ρ , and v in the right-hand part represent a sum of all modes:

$$v = v_1 + v_2 + v_3, \quad p = p_1 + p_2 + p_3, \quad \rho = \rho_1 + \rho_2 + \rho_3. \quad (26)$$

All cross nonlinear-viscous terms are neglected, as well as higher order nonlinear ones. If the nonlinear terms of the first mode are kept only, we get the known Burgers evolution equation, going to the famous Earnshaw one in the limit $\beta \rightarrow 0$:

$$\frac{\partial \rho_1}{\partial t} + \frac{\partial \rho_1}{\partial y} + \varepsilon \frac{-Q - S + 1}{2} \rho_1 \frac{\partial}{\partial y} \rho_1 - \frac{\beta}{2} \frac{\partial^2 \rho_1}{\partial y^2} = 0. \quad (27)$$

Physically, this case relates to a dominant rightward acoustic mode. For an ideal gas, again, $Q = -\gamma = -C_P/C_v$,
 $S = 0$.

4.2 Dynamics of beams.

In the same way, the famous Khokhlov-Zabolotskaya equation [14,15] (KZ, in the limit $\beta \rightarrow 0$) and Khokhlov-Zabolotskaya-Kuznetsov equation (KZK) for rightward (leftward) propagating beams follows when acting by P_1, P_2 at the system (7):

$$\frac{\partial \rho_{1,2}}{\partial t} \pm \frac{\partial \rho_{1,2}}{\partial y} \pm \frac{\mu}{2} \int \Delta_{\perp} \rho_{1,2} dy - \frac{\beta}{2} \frac{\partial^2 \rho_{1,2}}{\partial y^2} \pm \varepsilon \frac{-Q - S + 1}{2} \rho_{1,2} \frac{\partial}{\partial y} \rho_{1,2} = 0, \quad (28)$$

This evolution equation may be understood as a limit for the progressive mode self-action, meaning that only this mode nonlinear terms are kept. It seems be suitable definition though the 'self-action' is usually reserved for the phenomenon of acoustic heating when acoustic mode causes the entropy mode during its propagation thus changing the surrounding [8].

5 ACOUSTIC STREAMING CAUSED BY IMPULSES.

There are some reasons to find another approach to acoustic streaming. First, the procedure of temporal averaging is a basis of the modern theory of acoustic streaming and all corresponding results relate to averaged fields. All non-periodic acoustic sources thus are left of account. Second, the problem of modes interaction looks more extended. Actually there exist three types of independent modes. The initial amplitudes of these modes are determined by initial conditions of a concrete problems and may be evaluated by projecting of the overall initial field into every modal field. The acoustic streaming imposes an ability of predominate acoustic mode. The acoustic field causes slow varying with time flow which may grow with time since nonlinear effects can store, that is namely the acoustic streaming. So, the acoustic streaming relates to one type (though important) of possible nonlinear interactions in flow, namely to the beginning of acoustic mode evolution. Later, an amplitude of the vortical mode (and the entropy one also) may become so large that one can not treat these modes as secondary and other approach for nonlinear interaction of modes should be developed. Let us mark, that even traditional subdivision of overall flow into 'slow' and 'quick' components eliminates the entropy mode unclearly. It is also 'slow' but essentially possesses non-zero density perturbation which slowly varies with time ($\partial \rho_3 / \partial t = 0$ in linear approach), so the procedure of temporal averaging of continuity equation fails with storing of nonlinear effects. Also, there are many more complicated flows over inhomogeneous media and/or with background flows that may be algorithmically solved by the pointed method, see conclusion.

We will show, at first, that projecting results in the well-known equations for an acoustic streaming in terms of \vec{V} , and, the second and more important, that there appear the new equations for acoustic streaming caused by impulse (non-periodic) acoustic signals.

To consider interactions of vortical and acoustic modes, one acts by projector $P_4 + P_5$ at the system (7) to get an evolution equation for vorticity $\varphi_4 + \varphi_5$. The right-hand nonlinear vector involves, again, all modes inputs. Then, to calculate the nonlinear interactions of only vortical and rightward propagating acoustic mode, the right-hand vector should be thought as a sum of vortical and the first acoustic mode:

$$\psi = \begin{pmatrix} 1 - \frac{\mu}{2} \Delta_{\perp} \int dy \int dy - \frac{\beta}{2} \frac{\partial}{\partial y} \\ \sqrt{\mu} \frac{\partial}{\partial z} \int dy \\ 1 - (\delta_2^1 + \delta_2^2) \frac{\partial}{\partial y} \\ 1 \end{pmatrix} \rho_1 + \begin{pmatrix} v_{x4} \\ v_{y4} \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ v_{y5} \\ v_{z5} \\ 0 \\ 0 \end{pmatrix}. \quad (29)$$

An operator $\Delta_{\perp} \int dy \int dy$ represents in \vec{x} -space operator $\frac{k_x^2 + k_y^2}{k_y^2}$. Acting by $P_4 + P_5$ at the system (5) leads to an evolution equation (27) for vorticity $\varphi(\vec{x}, t)$, which we rewrite going to velocity components $V_x = v_{x4} = -\partial \varphi_4 / \partial y$, $V_y = v_{y4} + v_{y5} = \sqrt{\mu} \partial \varphi_4 / \partial x + \sqrt{\mu} \partial \varphi_5 / \partial z$, $V_z = v_{z5} = -\partial \varphi_5 / \partial y$:

$$\frac{\partial V_x}{\partial t} - \delta_1^2 \frac{\partial^2 V_x}{\partial y^2} = -\varepsilon (\vec{V} \nabla) V_x + \varepsilon F_{1x} \quad (30)$$

where in right-hand side of (30) only two term are left: the first relating to the vortical mode, and the second F_1 consisting of quadratic inputs of the first progressive acoustic mode:

$$F_{1x} = \begin{pmatrix} 1 - \mu \frac{\partial^2}{\partial x^2} \int dy \int dy & -\sqrt{\mu} \frac{\partial}{\partial x} \int dy & -\mu \frac{\partial^2}{\partial x \partial z} \int dy \int dy & 0 & 0 \end{pmatrix} (\varphi_1 + \varphi_{1tv}), \quad (31)$$

where φ_1, φ_{1tv} are supposed to consist of specific perturbations for the rightward dominant acoustic mode, see (10). For example, φ_1 looks

$$\varphi_1 = \begin{pmatrix} -(\vec{v}_1 \vec{\nabla})v_{1x} + \sqrt{\mu}\rho_1 \partial p_1 / \partial x \\ -(\vec{v}_1 \vec{\nabla})v_{1y} + \rho_1 \partial p_1 / \partial y \\ -(\vec{v}_1 \vec{\nabla})v_{1z} + \sqrt{\mu}\rho_1 \partial p_1 / \partial z \\ [Qp_1 + S\rho_1](\vec{\nabla} \vec{v}_1) - (\vec{v}_1 \vec{\nabla})p_1 \\ -\rho_1(\vec{\nabla} \vec{v}_1) - (\vec{v}_1 \vec{\nabla})\rho_1 \end{pmatrix}. \quad \text{There are all other possible cross nonlinear terms in the right-}$$

hand side of (30) which are out of interest in the present investigation since the rightward acoustic mode supposed to be dominant at least at the initial stage of the evolution. Writing on the terms of order not higher than $\beta\mu$, one goes to the expected result:

$$\begin{pmatrix} 1 - \mu \frac{\partial^2}{\partial x^2} \int dy \int dy & -\sqrt{\mu} \frac{\partial}{\partial x} \int dy & -\mu \frac{\partial^2}{\partial x \partial z} \int dy \int dy & 0 & 0 \end{pmatrix} \varphi_1 = 0, \quad (32)$$

that means that acoustical streaming may exist only in the thermoviscous flow. Finally, F_{1x} is as follows

$$\begin{aligned} F_{1x} = & \begin{pmatrix} 1 - \mu \frac{\partial^2}{\partial x^2} \int dy \int dy & -\sqrt{\mu} \frac{\partial}{\partial x} \int dy & -\mu \frac{\partial^2}{\partial x \partial z} \int dy \int dy & 0 & 0 \end{pmatrix} \varphi_{1tv} = \\ & \sqrt{\mu} \left[\frac{\beta}{2} \left(\frac{\partial \rho_1}{\partial x} \frac{\partial \rho_1}{\partial y} - \frac{\partial}{\partial x} \int dy (\rho_1 \frac{\partial^2 \rho_1}{\partial y^2}) - \frac{\partial}{\partial x} \int dy (\frac{\partial \rho_1}{\partial y})^2 \right) + (\delta_2^1 + \delta_2^2) \left(\frac{\partial}{\partial x} \int dy (\rho_1 \frac{\partial^2 \rho_1}{\partial y^2}) - \rho_1 \frac{\partial^2 \rho_1}{\partial x \partial y} \right) \right] + \\ & \sqrt{\mu} \left[\beta \left(\frac{\partial}{\partial x} \int dy \left(\rho_1 \frac{\partial^2 \rho_1}{\partial y^2} \right) - \rho_1 \frac{\partial^2 \rho_1}{\partial x \partial y} \right) + (\delta_1^1 + \delta_1^2) \left(\frac{\partial}{\partial x} \int dy (\rho_1 \frac{\partial^2 \rho_1}{\partial y^2}) - \rho_1 \frac{\partial^2 \rho_1}{\partial x \partial y} \right) \right]. \end{aligned} \quad (33)$$

To show that both effects play role: (1) presence of the thermoviscous terms in ψ_1 , and (2) presence of thermoviscous terms in φ_{1tv} , corresponding terms are included in square brackets, first and second ones though they partially compensate each other. Finally, (33) goes to the following:

$$F_{1x} = \sqrt{\mu}\beta \left(\frac{\partial \rho_1}{\partial x} \frac{\partial \rho_1}{\partial y} - 0.5\rho_1 \frac{\partial^2 \rho_1}{\partial x \partial y} - \frac{\partial}{\partial x} \int dy (\frac{\partial \rho_1}{\partial y})^2 \right), \quad (34)$$

and the constant of integration of the last term should be chosen accordingly to the real initial perturbations and/or boundary regime. It is remarkable that only three first elements of φ_{1tv} participate in the final radiation force, namely that three ones are responsible for the losses of momentum in the flow. The forth element expresses the thermoviscous loss of energy. This term (and also the fifth one) plays role in the forming of the heating, see the last line of \tilde{P}_3 given by (20). Relations of the specific perturbations of eigenvector ψ_1 (14) are used to express F_{1x} through ρ_1 .

The transverse component of the force caused by sound F_{1x} given by formula (34) does not possesses terms depending on the other transverse coordinate z because these terms are of order $\beta\mu^{3/2}$. In all calculations, the properties of specific modes and related projectors applied only. The temporal averaging was not proceeded. It has been proved that the acoustic force should be rotational [16]. In the referred papers, the rotational part of the averaged is just selected from the general expression. Projectors P_4, P_5 due to their properties yield in the vortical forces automatically.

5.1 Limit of the quasi-periodic acoustic source

To compare formula (30) with this caused by periodic at the tranducer source, let us take the transverse acoustic force (radiation force) given by Gusev, Rudenko [17]

$$\Phi_{1x} = \sqrt{\mu}\rho_0^2 \frac{\partial}{\partial x} (\theta^2/2) \exp(-\beta y) \quad (35)$$

rewritten in our variables; note also that longitudinal coordinate there is x instead of y . The radiation force (35) relates to the streaming evolving in the sound field of a highly attenuated beam with a quasi-periodic source as follows:

$$\rho_1(x, y, t) = \rho_0 \theta(x) \exp\left(-\frac{\beta}{2}y\right) \sin(t - y) \quad (36)$$

see [12] for more details. Calculating based on formula (34) yields in the following:

$$F_{1x} = \sqrt{\mu} \rho_0^2 \frac{\partial}{\partial x} (\theta^2/2) \exp(-\beta y) \left[1 + \frac{\beta}{4} \sin(2(t - y)) + O(\beta^2) \right] \quad (37)$$

Averaging over integral number of period of the sound wave in the leading order gives $\langle F_{1x} \rangle = \Phi_{1x}$. In contrast to the known results, (34) fits to calculate force caused by any acoustic source including non-periodic ones. Moreover, evolution of streaming given by temporal averaging is not suitable in principle for detail tracing with small temporal step because of the very initial procedure of averaging over time whose interval is much longer than a period of a sound wave.

5.2 Radiation force caused by mono-polar acoustic source

As an appropriate acoustic source satisfying the limitations of [17], let us take a mono-polar two-dimensional wave as follows [18]:

$$\rho_1(x, y, t) = -\sqrt{\frac{2\beta}{\pi}} \exp(-x^2) \frac{\exp(-\tau^2/2\xi)}{\epsilon \sqrt{\xi/\beta} (C - \text{Erf}(\tau/\sqrt{2\xi}))}, \quad (38)$$

where $\xi = \beta y$, $\tau = t - y$, $\epsilon = \frac{-Q-S+1}{2}$ is a parameter of nonlinearity, see (5). For ideal gases, $\epsilon = \frac{\gamma+1}{2}$. Constant C is responsible for the shape of the curve: large C provides a curve close to the Gauss one. The function F_{1x} achieves negative minimum at the transversal distance $x_m = \sqrt{2}/2$ and is an odd function as follows from (34), (38). Results of the calculating of $\frac{F_{1x}}{\sqrt{\mu\beta}}$ in accordance to formula (34) are shown at the figures 1,2 at the transversal point $x = x_m$. Figure 1 shows the dependence of $\frac{F_{1x}}{\sqrt{\mu\beta}}$ on y at $t = 1$, and figure 2 shows the same at $t = 3$. Constants of mono-polar source (35) used in calculations are: $C = 2$, $\epsilon = 1.2$, $\beta = 0.1$. Also, the constant of numerical integration of the last term of (31) is chosen to the radiation force be zero at $y \rightarrow \infty$.

All pictures show that streaming develop with some delay after the source passing. The reason is non-local relation between the source and induced streaming. The velocity field may be calculated accordingly to the formula (30) completed with equations for the other components of \vec{V} . At the first stage of evolution, the nonlinear term $(\vec{V} \cdot \nabla) \vec{V}$ may be neglected and the linear equation like this of thermal conductivity with viscous coefficient caused by shear viscosity δ_1^2 should be solved. The acoustic source in the right-hand side of (30) is namely the radiation force. The temporal behavior of the radiation force at the two longitudinal coordinates $y = 1$ and $y = 3$ is presented at the figures 3,4, $x = \sqrt{2}/2$. The mono-polar source is naturally attenuated during its propagation along y-axis. Note that the radiation force is multiplied by the large value $\frac{1}{\sqrt{\mu\beta}}$ at all pictures, so the real scale is much less.

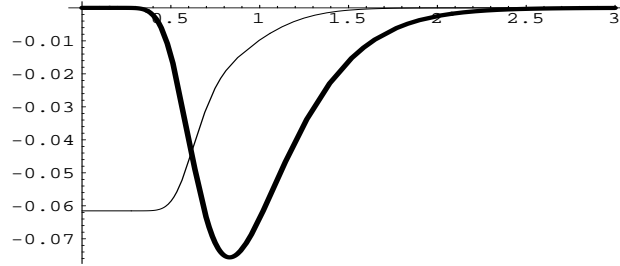


Fig.1 Longitudinal distribution of the radiation force $\frac{F_{1x}}{\sqrt{\mu\beta}}(y)$ (thin line) and the acoustic source $\rho_1(y)$ (bold line) at $t = 1, x = \sqrt{2}/2$.

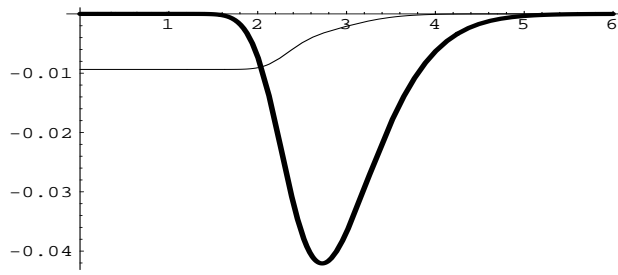


Fig.2 Longitudinal distribution of the radiation force $\frac{F_{1x}}{\sqrt{\mu\beta}}(y)$ (thin line) and the acoustic source $\rho_1(y)$ (bold line) at $t = 3, x = \sqrt{2}/2$.

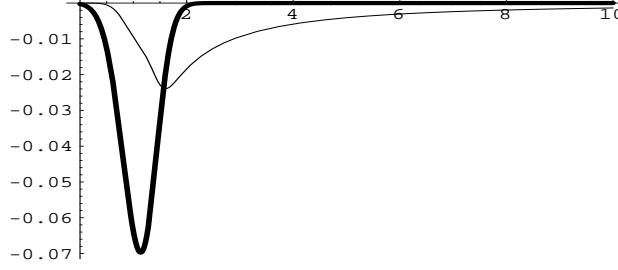


Fig.3 The temporal distribution of the radiation force $\frac{F_{1x}}{\sqrt{\mu\beta}}(t)$ (thin line) and the acoustic source $\rho_1(t)$ (bold line) at $y = 1, x = \sqrt{2}/2$.

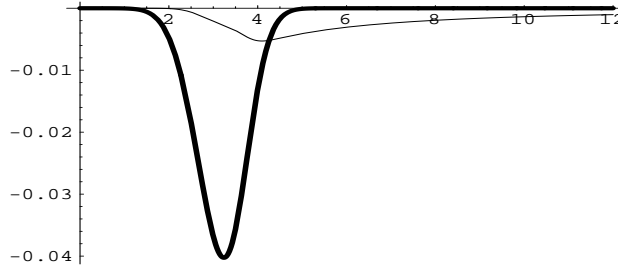


Fig.4 The temporal distribution of the radiation force $\frac{F_{1x}}{\sqrt{\mu\beta}}(t)$ (thin line) and the acoustic source $\rho_1(t)$ (bold line) at $y = 3, x = \sqrt{2}/2$.

6 Conclusions

The basic idea of the paper is to separate modes accordingly to their properties in the weakly nonlinear flow. At first, modes as eigenvectors of the linear flow should be defined. In the other words, the relations of the specific perturbations inside every mode should be established. Both homogeneous and inhomogeneous backgrounds (see paper on interacting modes in bubbly liquid [10]), media affected by external forces including the gravitational one which changes the background density and pressure [11] are treated in the algorithmic way. The definition of modes is unique, determined by the linearized differential conservation equations only. Proper initial or/and boundary conditions are given by the proper superposition of modes.

The second step is to get coupled nonlinear equations for the interacting modes and to solve it approximately. If one of the acoustic modes is dominant the generation of the entropy mode and the other acoustic mode in the plane geometry (namely heating and the reflected wave) is governed by the simple analytical formulae. The evolution equations may be corrected up to the higher order nonlinear terms due to increasing influence of the other generated modes [10,12].

The principal advance in comparison to the widely used approaches consists in possibility to treat any sources: initial mixture of all possible types of motion, non-periodic acoustic source and so on. In the present

paper, radiation force caused streaming is calculated for any acoustic source, an example of the monopolar pulse source is considered, the limit of periodic source is traced.

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